Bayesian Methods for ^{14}C Neutron Capture

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Introduction

Relevance

- Capture reaction ^{14}C(n, γ)^{15}C controls the carbon-nitrogen-oxygen cycle in the helium-burning regions of stars [1]
- Coulomb dissociation [1] and direct capture [2] data are compared with theory predictions
- Effectiveness of multiple EFT expansions are tested

Research Objectives

- The ^{14}C(n, γ)^{15}C cross section is calculated, and a systematic expansion is obtained [3]
- Ambiguity regarding parameters sizes leads to multiple theoretical expressions
- Bayesian analysis is applied to 1) determined unknown parameters and 2) compare the evidences for each theoretical expression

Cross Section

The ^{14}C(n, γ)^{15}C reaction rate is reflected in the cross section:

$$\sigma(p) = \frac{1}{2}\frac{64\pi\alpha}{M_c^2\mu^2}\frac{p\gamma(p^2+\gamma^2)}{1-\rho\gamma}\left[2\left|g^{^2P_{1/2}}(p)\right|^2 + 4\left|g^{^2P_{3/2}}(p)\right|^2\right],$$

where

$$g^{^2P_{3/2}}(p) = \frac{\mu}{p^2+\gamma^2} + \frac{6\pi\mu}{1/a_1^{(2)} + r_1^{(2)}p^2/2 - s_1^{(2)}p^4/4 - ip^3}\left[\frac{\gamma}{4\pi} + \frac{ip^3-\gamma^3}{6\pi(p^2+\gamma^2)}\right],$$

unknown parameters in red [3]

- The cross section is expressed as an expansion in Q/Λ:
  - Low-momentum scale Q~40 MeV
  - High-momentum scale Λ~100-200 MeV
- Two different models derived based on different sets of assumptions (Table 1)
- LO and NLO expansions for each model

Table 1. Parameter sizes

Model A	Model B
$\rho \sim \frac{1}{\Lambda}$	$\rho \sim \frac{1}{Q}$
$a_1^{(2)} \sim \frac{1}{Q^3}$	$a_1^{(2)} \sim \frac{1}{\Lambda Q^2}$
$r_1^{(2)} \sim Q^3$	$r_1^{(2)} \sim \Lambda$
$s_1^{(2)} \sim \frac{1}{\Lambda}$	—

Results

Fig 1. ^{14}C(n, γ)^{15}C capture cross section

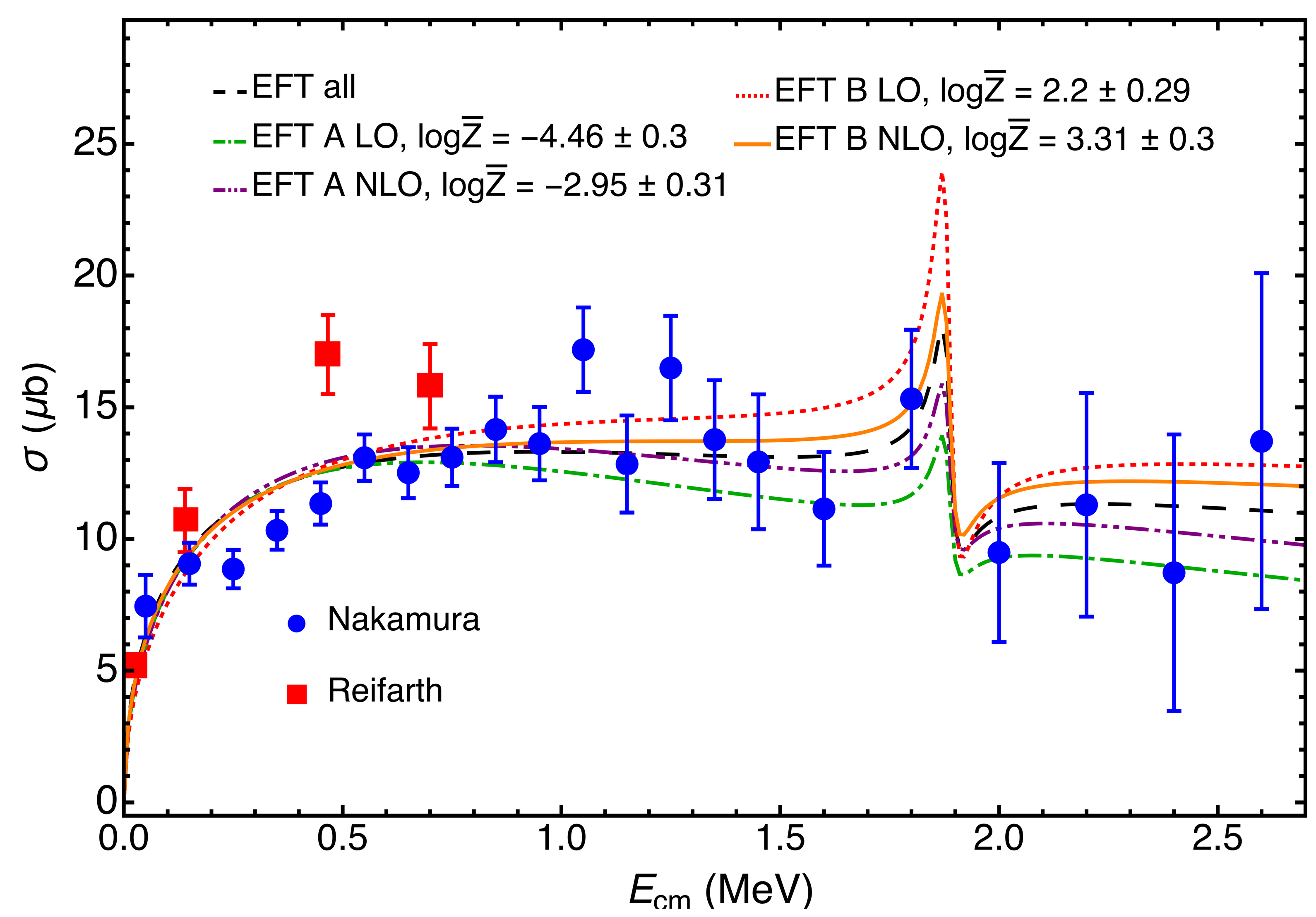
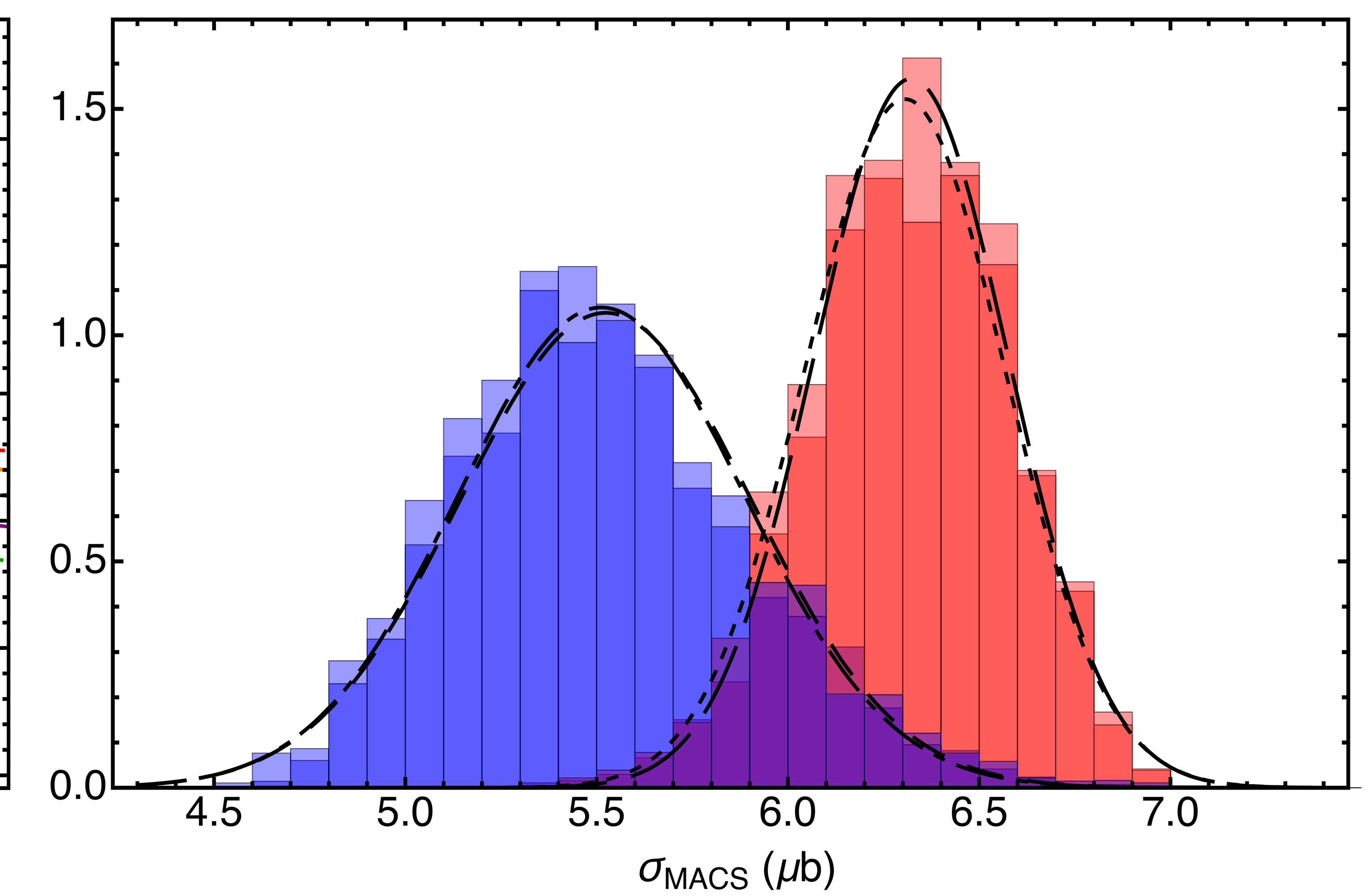


Fig 2. MACS cross section with (right) and without (left) 23.3 keV data



- Optimal parameters and log-evidence (log$\bar{Z}$) calculated
- Nested Sampling implemented using Python Nestle library [6, 7]
- Bayesian likelihood calculated as deviation between data and theory
- Maxwellian average cross sections (MACS) at 23.3 keV: reaction rate scaled by Maxwell-Boltzmann velocity distribution [8]
- Data at 23.3 keV is omitted in some runs; MACS calculations without this data yield smaller values

Bayesian Methods

Bayes' Theorem

$$\underbrace{p(\theta|D)}_{\text{Posterior}} = \frac{\overbrace{p(\theta)}^{\text{Prior}}\ \overbrace{p(D|\theta)}^{\text{Likelihood}}}{\underbrace{p(D)}_{\text{Evidence}}}$$

- Priors incorporate theoretical assumptions

Evidence Calculation

$$Z = p(D) = \int p(\theta)p(D|\theta)d\theta$$

- Bayesian methods favor simplicity of models, penalizing extra parameters (Occam's razor) to enhance generalizability of predictions [4]

Conclusion & Future Works

- Nested sampling for optimal parameters and log$\bar{Z}$
- Evidence favors Model B over Model A
- More precise data at higher energy may reveal differences between B-LO and B-NLO
- Future work includes Bayesian analysis of proton capture on Beryllium

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